Project Idea: The Fugitive

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Introduction

A fugitive is on the run from the police. He finds a hiding spot underground in a particular location. He intends to stay in this hiding spot until the police officers arrive, search the surrounding area, and leave to look somewhere else. However, he must periodically come up for air, or else he will suffer from oxygen deprivation. Coming up for air temporarily exposes him, thereby putting him at the risk of being caught by the authorities. The goal is to devise a strategy that will minimize the combination of his chance of being caught by the police and the damage caused by suffocating.

The model

Here are the parameters and variables of the model:

- The fugitive does not know when the police will arrive; their arrival time is a random variable T with probability density function $f_T(t)$. (Realistically, this might be exponentially distributed.) When the police officers do finally arrive, they leave immediately if they cannot find the fugitive (i.e. if he is hiding underground), and the game is then over (i.e. the fugitive sees the police leave, and knows he is then safe to leave his hiding spot and run free).
- The fugitive wants to minimize his oxygen deprivation, which imposes an incremental penalty described by the function $s(t)$ of the last time he was above ground. $s(t)$ is monotonically increasing.
- Going up for air takes constant time h, during which the fugitive is exposed to being caught.
- Getting caught incurs a penalty of c.
- The fugitive's decision variable is described by the vector b, whose entries b_i represent the time at which the fugitive comes up for his i^{th} breath. *Note: if this problem goes on for infinite time, the vector would be infinitely long, which is of course a problem. In my final version of the model I might make this a finite-time problem or discretize time.*
- Here is our objective function:

$$\min_b \quad \underbrace{\int_0^{b_1} s(t)dt}_{\text{Damage until first breath}} \quad + \quad \underbrace{\sum_{i=1}^{\infty} \int_{b_i+h}^{b_{i+1}} s(t-t_i)dt}_{\text{Damage between ensuing breaths}} \quad +c \quad \underbrace{\sum_{i=1}^{\infty} \int_{b_i}^{b_i+h} f_T(t)dt}_{\text{Probability of being caught}}$$

Thoughts/questions

- How can this be modeled in a simple way? For instance, are there particular probability distributions for the police's arrival time that make this problem easy to solve? If the probability distribution is memory-less, like the exponential distribution, it might be easy to solve this problem, since the problem "restarts" after each breath. I could use a greedy algorithm, simply ignoring all future breaths and maximizing my immediate utility (i.e. go breathe as soon as the marginal damage of waiting exceeds the marginal damage of exposing myself to being caught). This might not be perfectly accurate, but if it does not alter my results dramatically it might be a good basic strategy.

- It seems tough to get a general analytical solution, so I might just come up with some plausible strategies (e.g. maximize immediate benefit), and see which works best in a simulation.